CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

[Turn over

2

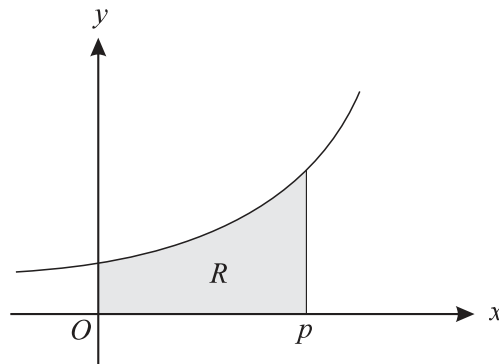**1** Solve the inequality $|x - 4| > |x + 1|$. [4]

2 The polynomial $x^4 - 9x^2 - 6x - 1$ is denoted by $f(x)$.

   (i) Find the value of the constant a for which

$$f(x) \equiv (x^2 + ax + 1)(x^2 - ax - 1).$$ [3]

   (ii) Hence solve the equation $f(x) = 0$, giving your answers in an exact form. [3]

3

The diagram shows the curve $y = e^{2x}$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$.

   (i) Find, in terms of p, the area of R. [3]

   (ii) Hence calculate the value of p for which the area of R is equal to 5. Give your answer correct to 2 significant figures. [3]

4   **(i)** Show that the equation

$$\tan(45° + x) = 4\tan(45° - x)$$

   can be written in the form

$$3\tan^2 x - 10\tan x + 3 = 0.$$ [4]

   (ii) Hence solve the equation

$$\tan(45° + x) = 4\tan(45° - x),$$

   for $0° < x < 90°$. [3]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has exactly one root. [3]

(ii) Verify by calculation that the root lies between 1.0 and 1.4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

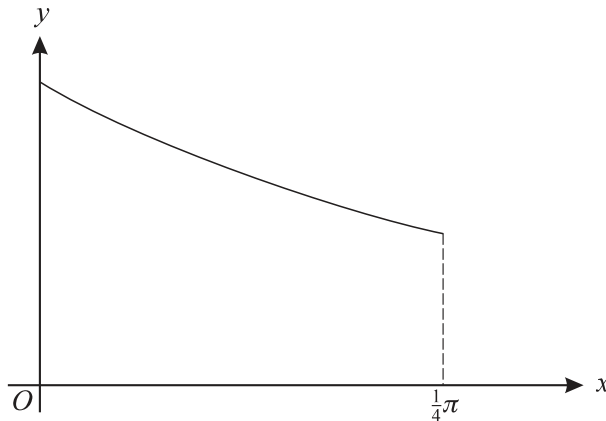**6** The equation of a curve is $y = \dfrac{1}{1 + \tan x}$.

(i) Show, by differentiation, that the gradient of the curve is always negative. [4]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 + \tan x} \, dx,$$

giving your answer correct to 2 significant figures. [3]

(iii)

The diagram shows a sketch of the curve for $0 \leqslant x \leqslant \frac{1}{4}\pi$. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

7 The parametric equations of a curve are

$$x = 2\theta - \sin 2\theta, \quad y = 2 - \cos 2\theta.$$

(i) Show that $\dfrac{dy}{dx} = \cot \theta$. [5]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$. [3]

(iii) For the part of the curve where $0 < \theta < 2\pi$, find the coordinates of the points where the tangent is parallel to the x-axis. [3]

4

BLANK PAGE